$$+\frac{2(2b^2c^2+2a^2b^2-3a^2c^2-b^4)m^2}{(a^2-b^2)(b^2-c^2)(C^2-2a^2-2b^2-2c^2)}$$

$$+\frac{2(c^4-2a^2c^2-2b^2c^2+3a^2b^2)n^2}{(a^2-c^2)(b^2-c^2)(C^2-2a^2-2b^2-2c^2)}=1.$$

$\therefore\ l^2/R^2+m^2/S^2+n^2/T^2=1$, a co-axial ellipsoid.

CALCULUS.

89. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

Integrate the equation, $\dfrac{dy}{dx}+y\cos x=\dfrac{\sin 2x}{2}$.

I. Solution by Dr. E. D. ROE, Jr., Norwood, Mass.

Put $\sin x-y=z$; then $\cos x dx-dy=dz$, $dy=\cos x dx-dz$, and the equation becomes

$$\frac{dz}{dx}+(z-1)\cos x=0, \text{ or } \frac{dz}{z-1}+\cos x dx=0.$$

Integrating this, $\log(z-1)+\sin x+\kappa=0$, $z-1=e^{-\sin x-\kappa}$,

$$\text{or } \sin x-y-1=e^{-\sin x-\kappa}=-ce^{-\sin x},\ y=\sin x-1+ce^{-\sin x}.$$

Dr. Roe also furnished a second solution.

II. Solution by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, O.

If the equation $\dfrac{dy}{dx}+y\cos x=0$ is solved, the result is $y=C_1e^{-\sin x}$. After substituting in the original equation $\dfrac{dC_1}{dx}$ is found to equal $e^{\sin x}\sin x\cos x$; therefore, $C_1=e^{\sin x}\sin x-e^{\sin x}+C$. And $y=\sin x+Ce^{-\sin x}$.

III. Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass., and JOHN R. JEFFERY, Student in Ohio State University, Columbus, O.

The general form of this equation is $\dfrac{dy}{dx}+Py=Q$ of which the general integral is $e^{\int Pdx}y=\int e^{\int Pdx}Qdx+C$. Here $\int Pdx=\int\cos x dx=\sin x$.

$\therefore\ e^{\sin x}y=\int e^{\sin x}\cos x\sin x dx+C$.

Integrating right member by parts, $e^{\sin x}y=\sin x.e^{\sin x}-e^{\sin x}+c$, or $y=\sin x-1+ce^{-\sin x}$.

[See Johnson's *Differential Equations*, page 35, ex. 7.]

IV. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics in Manual Training School, Philadelphia, Pa.; M. C. STEVENS, M. A., Professor of Mathematics, Purdue University, Lafayette, Ind.; and BURKE SMITH, Senior Class, University of Washington, Seattle, Wash.

$\dfrac{dy}{dx}+y\cos x=\sin x\cos x.$

Multiply by $e^{\sin x}$ as an integrating factor.

Then $e^{\sin x}dy+y\cos x.e^{\sin x}dx=e^{\sin x}\sin x\cos x dx.$

Integrating, $y.e^{\sin x}=\int e^{\sin x}\sin x\cos x dx.$

Integrate right hand member by parts, and we have,

$$ye^{\sin x}=e^{\sin x}\sin x-\int e^{\sin x}\cos x dx=e^{\sin x}\sin x-e^{\sin x}+c.$$

$\therefore\ y=\sin x-1+ce^{-\sin x}.$

V. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $\sin x=\log v$; so that $\cos x dx=dv/v$. Then the equation becomes $dy/dv+y/v=\log v/v$.

$\therefore\ vdy+ydv=\log v dv.\quad \therefore\ vy=v(\log v-1)+C.\quad \therefore\ y+1=\log v+C/v.$

$\therefore\ y+1=\sin x+Ce^{-\sin x}$

VI. Solution by J. SCHEFFER, A. M., Hagerstown, Md., and ARTHUR C. LUNN, University of Chicago, 5630 Drexel Ave., Chicago, Ill.

Solving first the differential equation $\dfrac{\partial y}{\partial x}+y\cos x=0$, we find $y=Ce^{-\sin x}$; differentiating, we have $\dfrac{\partial y}{\partial x}=-Ce^{-\sin x}\cos x+e^{-\sin x}\dfrac{\partial C}{\partial x}.$

$$\therefore\ e^{-\sin x}\frac{\partial C}{\partial x}=\tfrac{1}{2}\sin 2x.\quad \therefore\ C=\tfrac{1}{2}\int e^{\sin x}\sin 2x.dx.$$

Putting $\sin x=z$, $\therefore\ \cos x=\sqrt{1-z^2}$, $\partial x=\dfrac{\partial z}{\sqrt{1-z^2}}$; we have,

$$C=\int e^z.z\partial z=\int z\partial(e^z)=ze^z-\int e^z\partial z=ze^z-e^z=\sin x.e^{\sin x}-e^{\sin x}+c.$$

Substituting this in $y=Ce^{-\sin x}$, we obtain $y=\sin x-1+ce.$

The method employed is Lagrange's method of variation of parameters.

Also solved by *HENRY HEATON*, and *P. H. PHILBRICK*.